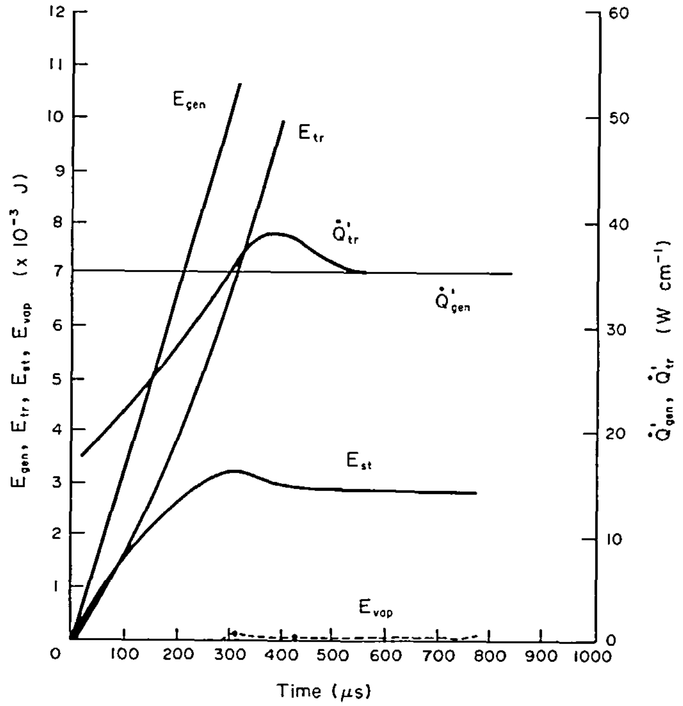

FIG. 4. Energy balance and heat transfer during the boiling transient shown in Fig. 2.

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HEAT TRANSFER FROM PARTIALLY INSULATED HEXAGONAL DUCTS

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NOMENCLATURE

D_h	equivalent diameter
f	friction factor
N	dimensionless distance normal to the duct wall
Nu_{H_1}	Nusselt number—constant axial heat flux, isothermal local periphery
Nu_{H_2}	Nusselt number—constant axial heat flux, uniform heat flux on the periphery at a given axial location
P	pressure
p	duct perimeter
Pr	Prandtl number
Re	Reynolds number
u	axial velocity
s	arc length
T	temperature
x, y, z	spatial coordinates

Subscripts

b	bulk
w	wall

1. INTRODUCTION

HEXAGONAL passages are the subject of some modern engineering problems. Examples can be quoted from the design of hexagonal compact exchangers [1] and from the study of friction and heat transfer characteristics of flow of molten glass in ceramic hexagonal conduits [2]. Fully developed laminar flow and heat transfer can be frequently encountered inside ducts having small hydraulic diameters or carrying highly viscous fluids. Shah and London [3] have compiled and described in detail the methods available in the literature for the solution of this class of problems. Using conformal mapping, Tao [4] analysed the case of the laminar fully developed flow with linear axial wall temperature distribution and arbitrary heat generation in the hexagonal ducts. Hsu [5] treated with finite difference the axially constant heat flux thermal boundary conditions with or without energy generation in the fluid. He considered two situations around the duct periphery. The first had uniform heat flux around the periphery and the second was the situation when the heat flux from each pair opposing sides was uniform and equal but different in magnitude from the fluxes between the other two pairs of sides. The reported Nu_{H_2} is 3.795. Employing the nine-point matching technique, Cheng [6] calculated the Nu_{H_1} in regular polygonal ducts with axial constant wall heat flux and isothermal periphery. Cheng [7] extended the above work to include viscous dissipation and uniform heat generation. Again, Cheng [8], with the more accurate twelve-point matching method, determined the Nu_{H_2} in the regular polygonal ducts. The Nu_{H_2} value for the hexagonal duct from Cheng [8] is 3.862, which is higher than that of Hsu [5] by 1.765%.

The objective of the present investigation is to generate theoretical Nusselt values for the fully developed laminar flow inside partially insulated hexagonal ducts receiving constant axial heating. In view of the thermal condition around the duct periphery at a given axial location, the thermal situation on the solid exposed boundary can be further classified. Two such subcategories are considered, namely, the isothermal and the uniform heat flux peripheries.

2. ANALYSIS

The constant property, laminar fully developed momentum and energy equations under the absence of viscous dissipation, energy generation within the fluid, axial conduction, and natural convection, when subjected to the following transformations:

$$U = -\frac{u}{u_b Re \dfrac{d\bar{P}}{dX}}, \quad \bar{P} = \frac{P}{\rho u_b^2}, \quad \Psi = \frac{T-\bar{T}_w}{4\left(\dfrac{qD_h}{K}\right) Re \dfrac{d\bar{P}}{dX}},$$

$$X = \frac{x}{D_h}, \quad Y = \frac{y}{D_h}, \quad Z = \frac{z}{D_h} \tag{1}$$

where $\bar{T}_w$ is the average wall temperature at a given axial location defined by

$$\bar{T}_w(x) = \frac{1}{p}\oint_p T_w(x,y,z)\,ds$$

yield the following dimensionless forms of the momentum and energy equations:

$$\frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} + 1 = 0, \tag{2}$$

$$\frac{\partial^2 \Psi}{\partial Y^2} + \frac{\partial^2 \Psi}{\partial Z^2} + U = 0. \tag{3}$$

By invoking an energy balance between the axial variation of the total enthalpy of the flowing fluid and the convected energy to the duct wall, it is possible to relate the Nusselt number to the axial temperature gradient by

$$\frac{4\,Nu}{Re\,Pr\,D_h} = \frac{1}{(T_b-\bar{T}_w)}\frac{dT_b}{dX}. \tag{4}$$

The above equation is valid for fully developed flow in a duct regardless of the type of the thermal boundary. The relation between the heat flux supplied uniformly along the duct axis and the bulk temperature axial gradient,

$$\frac{dT_b}{dx} = \frac{4q}{\rho c u_b D_h}, \tag{5}$$

when inserted in equation (4), gives a simple relation between the average Nusselt number and the bulk value of the dimensionless temperature Ψ,

$$Nu = \frac{1}{\Psi_b}. \tag{6}$$

Equation (6) has the same pattern as the one relating the friction factor and the average dimensionless velocity defined by the transformations (1)

$$f\,Re = \frac{2}{U_b}. \tag{7}$$

Boundary conditions

The boundary condition for the momentum equation (2) is that $U = 0$ on the solid boundary. As stated in the previous section, two thermal boundary conditions are investigated:

(1) Uniform axial heat flux and partially isothermal-partially insulated periphery at a given axial location, i.e.

$$\Psi = 0 \quad \text{and} \quad \frac{\partial \Psi}{\partial N} = 0. \tag{8}$$

The calculated Nusselt number in this case will be designated Nu_{H_1}.

(2) Uniform axial heat flux and partially heated by a uniform heat flux-partially insulated periphery; the conditions for equation (3) are

$$\frac{\partial \Psi}{\partial N} = -1 \quad \text{and} \quad \frac{\partial \Psi}{\partial N} = 0. \tag{9}$$

The Nusselt number is given the symbol Nu_{H_2} to distinguish it from the previous situation.

The linearity of the momentum part of the problem simplifies the method for the solution of its finite difference equation. The solution is straightforward and was accomplished by the Gauss–Seidel iterational procedure.

The asymmetry of the thermal boundary in some of the cases studied [Figs. 1 and 2] necessitated that the calculation domain cover the whole hexagonal duct. The number of the grid points on the y and z axes (Fig. 1) is 33 and 65, respectively. The integration of the dimensionless velocity and temperature profiles to determine the average velocity U_b and the bulk temperature Ψ_b is carried out numerically using a 2-dim. extension of the Simpson's rule [9]. The friction factor is calculated from equation (7), while the incremental pressure drop for complete flow development and the length of the developing flow are computed by substituting the numerically found velocity profile in the equations of Lundgren *et al.* [10] and McComas [11].

The iterative solution of the energy equation (3) starts by inserting the thus obtained velocity profile $U(Y,Z)$ and an arbitrarily selected initial guess of the dimensionless temperature distribution $\Psi(Y,Z)$ in the set of difference equations. The initial guess should satisfy either the boundary conditions in equations (8) or (9).

Performing a line integration over the duct boundary of the definition of the dimensionless temperature Ψ, equation (3), yields

$$\oint_p \Psi_w(Y,Z)\,ds = 0. \tag{10}$$

After each iteration, the value of the integral in the LHS of

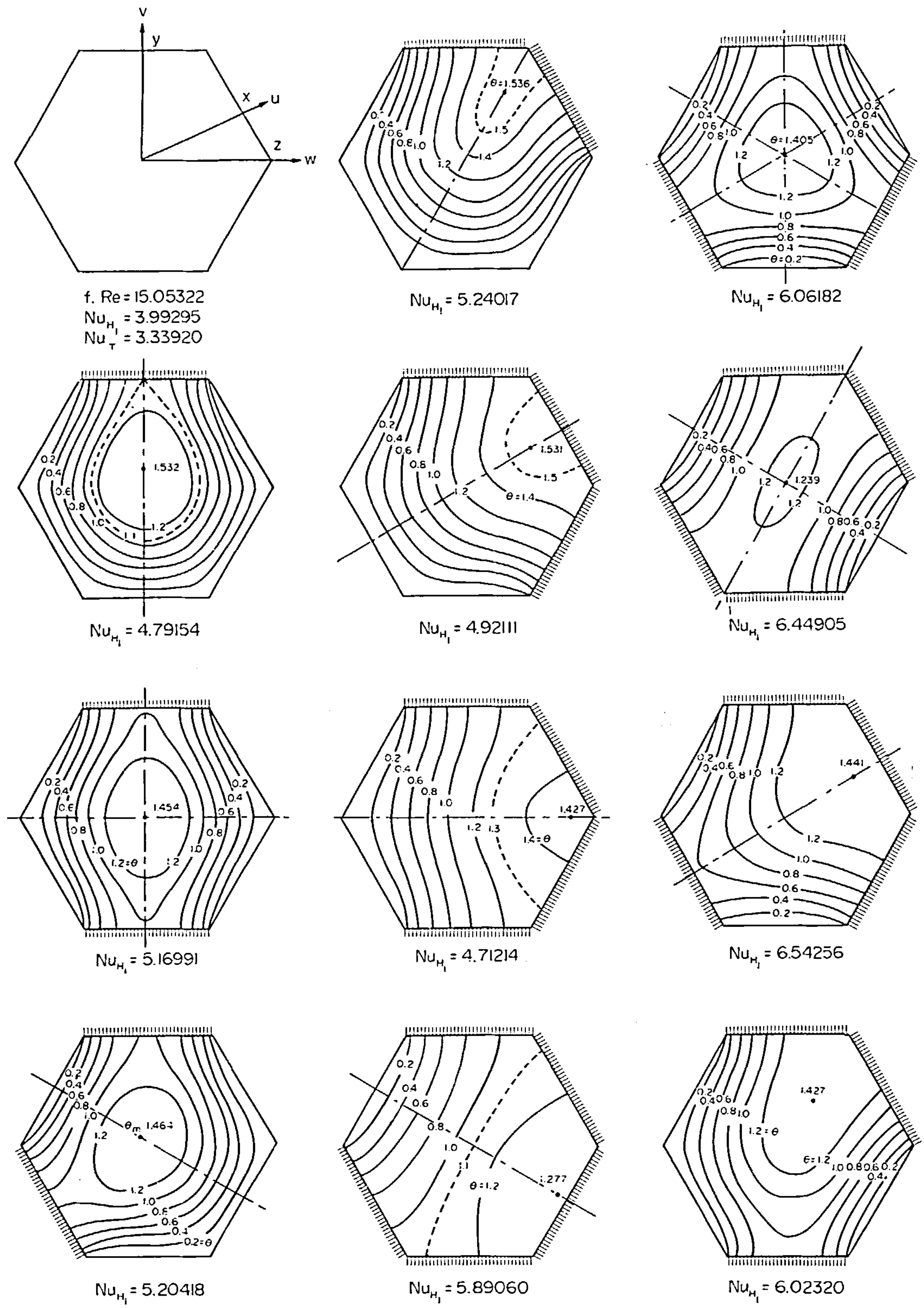

FIG. 1. Fully developed Nusselt numbers, Nu_{H_1}, in a partially insulated hexagonal duct.

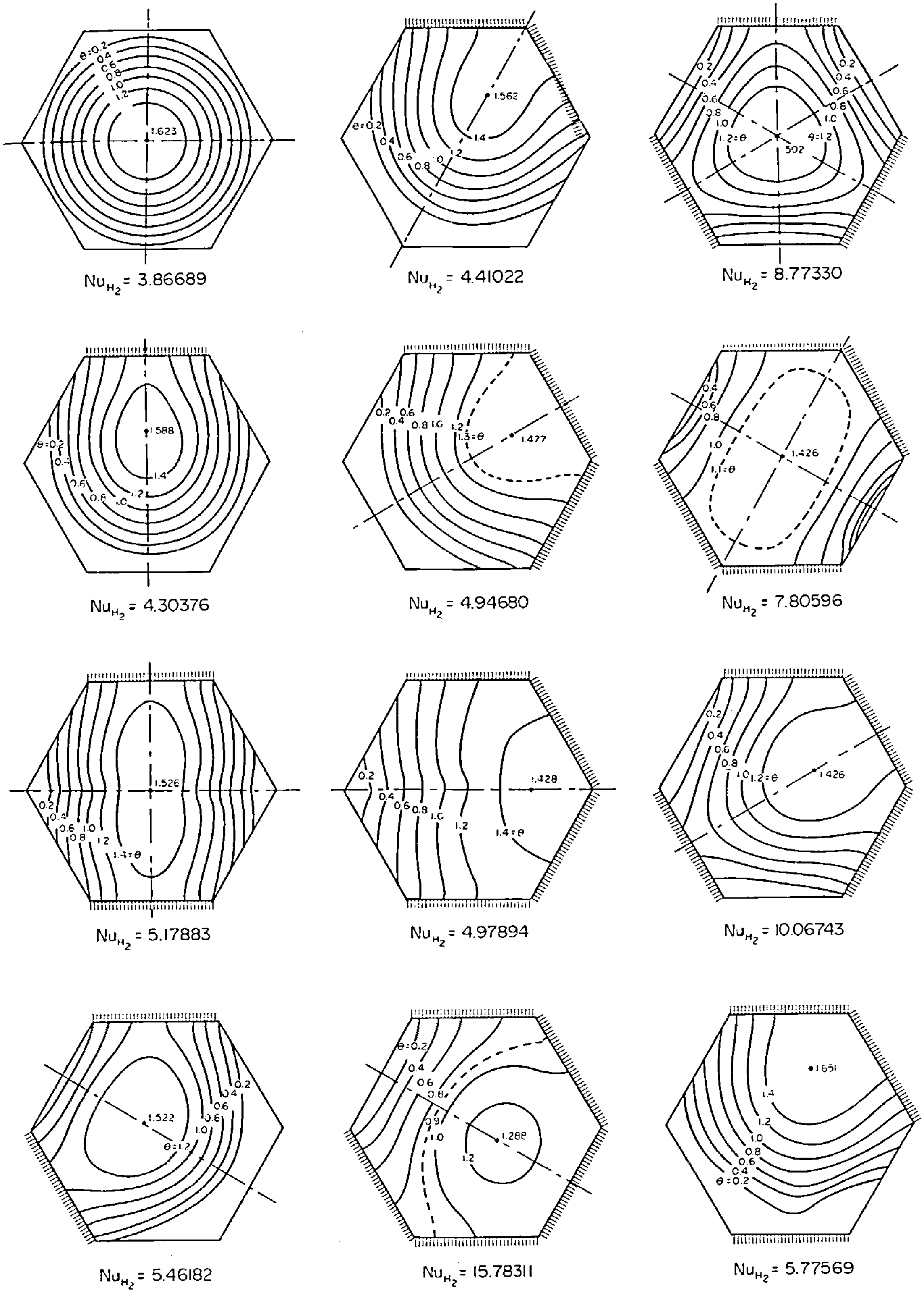

FIG. 2. Fully developed Nusselt numbers, Nu_{H_2}, in a partially insulated hexagonal duct.

equation (10) is determined. This amount represents the difference between the average wall temperature at a given iteration and the true average wall temperature, $\bar{T}_w$. Since all temperatures are referenced to the average wall temperature, $\bar{T}_w$, the convergence of the dimensionless temperature can be enhanced by subtracting after each iteration the residue of equation (10) from all the dimensionless temperatures on the wall and in the domain except those on the wall specified by $\Psi = 0$. The temperatures corrected for the average wall temperature are then used in the next iteration. After convergence has been achieved, the bulk temperature, Ψ_b, is evaluated and its inverse is the Nusselt number [equation (6)]. In presenting the heat transfer results, the characteristic length selected to form the Nusselt number is the hydraulic diameter. This is irrespective of the fact that a portion of the hexagon boundary is adiabatic and does not contribute in transferring heat to the fluid. This is a choice which maintains the consistency between the hydrodynamic and thermal sides of the problem.

3. RESULTS AND DISCUSSIONS

In order to assess the validity and accuracy of the finite difference computional procedure, the attention was first focused on one-quarter of the duct. The number of grid points selected on the Y and Z axes are also 65 and 33. Considering one-sixth of the duct might lead to better numerical accuracy for the same total number of grid points, but the form of the finite difference formulation will be different from that written for the total hexagonal duct owing to the existence of two inclined boundaries in the case of the sixth. The friction factor and the Nusselt numbers Nu_{H_1} and Nu_{H_2} for completely exposed hexagon are calculated using one-quarter of the duct. The same variables are then produced employing a full hexagon calculation domain. The results from one-quarter and the full hexagon as well as those obtained from the twelve-point matching method developed by Cheng [8] are compiled for comparison in Table 1.

Inspection of Table 1 reveals that the percentage difference in $f\,Re$, Nu_{H_1}, and Nu_{H_2} for the quarter and full hexagon calculations domains are 0.015, 0.17, and −0.038%, respectively. Confining the calculation domain to only one-quarter of the hexagon instead of the full duct does not materially alter the characteristic values of the flow and heat transfer. As long as the complete hexagonal domain is needed to handle the partially insulated ducts, which encompasses unsymmetrical cases, it is fruitful to compare the full duct results with those of Cheng [8]. A still closer difference of 0.005% exists between the hydrodynamic result of Cheng [8] and the numerical computations with the full hexagon. This improvement does not exist in Nu_{H_1} and Nu_{H_2} values and the deviations are wider and equal to +0.226% and −0.127%. The twelve-point matching method, due to Cheng [8], has the same nature as an analytic exact solution. Therefore, the results of Cheng [8] are believed to be accurate. The numerical output of the present investigation gives support to this statement, and the differences in Table 1 are probably due to the residual error in the finite-difference formulation. Consequently, the expected accuracies of the friction factor, Nu_{H_1}, and Nu_{H_2} of the partially insulated ducts presented in this work are approximately 0.005, 0.2, and 0.1%, respectively.

Table 1. Accuracy of computing the hexagon characteristics

	One-quarter 65 × 33	Full hexagon 65 × 33	Cheng [8]
$f\,Re$	15.05544	15.05322	15.054
Nu_{H_1}	3.9998	3.99295	4.002
Nu_{H_2}	3.8654	3.86689	3.862

After assessing the validity and accuracy of the finite difference computational procedure, the next task is to proceed and develop the Nusselt numbers for the partially insulated ducts. Eleven classifications from the point of view of the number of insulated sides and their relative positions cover all possibilities. Figures 1 and 2 are catalogues of the thermal results corresponding to the thermal boundary conditions in equations (8) and (9), respectively. For convenience, the value of the Nusselt number in each case is written below the sketch representing it. The Nusselt number based on the hexagon hydraulic diameter and not on a characteristic length based on the perimeter through which the heat flows, facilitates the comparison between the different cases and reflects directly the effectiveness of the average rate of heat transfer through a unit area of the exposed walls. Isothermal lines, Ψ = constant, are also plotted on the sketches. It is interesting to notice about the distribution of the isothermal lines that it seems as if they have a gas bubble in their middle which is attracted and distorted by the effect of the insulated surfaces acting in the same sense as free surfaces.

In order to complete the information about the fully developed laminar flow and heat transfer in hexagonal ducts, the hydrodynamic length needed for complete flow development, the incremental pressure drop in the entrance region, and the Nusselt number for an isothermal duct are calculated. The values obtained are 0.02854, 1.40714, and 3.3392, respectively. The Nu_T is determined using the solution method outlined in ref. [12] and is developed to handle the nonlinear energy equation in the case of isothermal ducts.

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